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MAKING USE OF LOGIC

Abstract. It seems that Polish logic has always been open to considerations concerning the use of methods and results of formal logic within disciplines. We overview a couple of such Polish contributions to what may be called the realm of applied logic. We take a closer look at the formalization of natural reasoning, inconsistency-tolerant logic, and at the formal analysis of causal nexus.

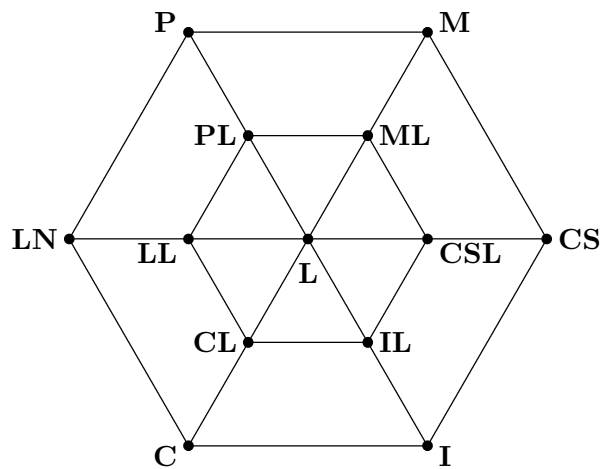
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1. Logic at work

In the following I shall outline the main Polish contributions to what is called “applied logic”. Naturally, the first question one might wish to have answered concerns the nature of applied logic itself. The usual mode of explication (*genus proximum* and the appropriate *differentiæ specificæ*) will not be feasible here, I’m afraid: it is plainly not sufficiently clear to me what *genus proximum* should be taken, in other words: what logic is. However, the *differentiæ specificæ* is perfectly clear: applied logic consists of results and methods of logical investigation put to work elsewhere, outside logic. Well, not everywhere outside logic — perhaps not in the history of Roman Literature nor in feminist pedagogics — but still in quite a few disciplines. Nowadays, the most interesting fields of application seem to be the following six areas, illustrated best perhaps by a slightly modified version of *Perzanowski’s cobweb*:



Logic, at the center, is surrounded by *Philosophy*, *Mathematics*, *Computer Sciences*, *Informatics*, *Cognitive Sciences*, and *Linguistics* (comp. Perzanowski 1993, 5). *Logic* is traditionally closely connected with mathematics and philosophy. At present, there is a strong relation to linguistics and further-

more to sciences which did not even exist in the times of Russell or Łukasiewicz. In all cases, as Perzanowski has put it, intermediate disciplines between logic and each of these sciences have emerged: *Philosophical Logic*, *Mathematical Logic*, *Logical Programming*, *Information Theory*, *Cognitive Logic*, and *Mathematical Linguistics*.

One might indeed wonder whether computer sciences and informatics should be treated as different branches, or what to make of psychology, artificial intelligence or physics. But the main idea is clear and convincing: logic is evidently not an isolated discipline. It is part of a large web of different sciences.

The leading idea in applied logic is to enlarge that realm of our language which allows precise strategies for controlling the correctness of arguments. If someone is inclined to apply the results and methods of logic, then she may choose one of two options. The first one is the traditional way of making use of logic. It is represented by the notorious magister teaching at the “Collegium Logicum” — as impressively described by Johann Wolfgang von Goethe in his “Faust”: by squeezing his pupils’ minds into Spanish boots, frustrating their imagination and invention. Or, in other words (though the above metaphor is obviously unfair to contemporary logic): the traditionalist doubts people’s competence in using their own language — he is the only one who can teach them to speak (and to think) correctly.

Or one may take another way: Martin Luther’s attitude, when translating the Bible: he frequented the market place in order to find out first how people do speak. He was perfectly conscious that the success of his translation depended on whether it would be accepted by his fellow citizens or not, i.e. depending on whether they recognized the terminology of the translation as their own language.

This is also the case with formalization in applied logic. Since logic has no means to enforce obedience to its linguistic settings, the success of the enterprise depends on how closely it fits the real terminology which it intends to formalize.

Often enough in history, the first way led to the opinion that logic is an esoteric discipline, useless and bothersome. For quite obvious reasons, this is a highly unwelcome and even dangerous image of logic. So it turns out that actually only the second option makes a good choice. It seems to me that Polish Logic is traditionally concerned with the question of the transdisciplinary applicability of its results. The reason for this state of affairs lies probably in its very origin as a transdisciplinary scientific enterprise. The main representatives of the Lvov-Warsaw School were more open than oth-

ers to scientific discourse and, consequently, interested in using the results of their logical work outside the narrow limits of this discipline.¹

As there is an area of application, most naturally the question appears what sort of logic can be used there. There are plenty of examples. I will concentrate on the following three.

2. Natural Deduction

Far from being precisely defined, the concept of natural deduction denotes a large class of approaches aiming at rule-based descriptions of logical calculi. This is not an invention of our days. Historians of logic suspect the genesis of natural deduction already in Aristotelian syllogistics and in Stoic logic where the deduction theorem was used in practice. In 1926, Łukasiewicz stated the problem of a formal reconstruction of theorem proving in mathematical praxis. These practical demonstrations of mathematical truths were far from what counted as a proof in logic. Proofs by assumptions, indirect proofs or alternative cases in the course of a proof all require theoretical elucidation. As a side effect of these investigations one might hope for a formal correct and more feasible method of proof in logic, since proving theorems within axiomatic systems is in most cases quite a hard job.

A common origin of almost all constructions intended to serve practical purposes (such as Mizar — an intelligent archives of all mathematical knowledge, what is an integral part of the world-wide research project QED) is Jaśkowski's considerations on natural deduction.

The alternative approach originating with Gerhard Gentzen aims rather at proof-theoretic aspects and is thus less interesting for application.

3. Parainconsistent Logic

In August 1997 the First World Congress of Paraconsistency was held in Ghent. It gave merit to the Polish line of research in this area, i.e. so-called non-adjunctive or discussive logic, invented fifty years ago, by Jaśkowski again. As the name indicates, formal contradictions are tamed by restricting the adjunction of propositions. (Not always is the adjunction of two true formulas a true formula.)

¹ This does not mean, of course, that new results of formal logic were judged on a basis of what applications do they have.

Jaśkowski was concerned with the logical treatment of inconsistent information as it occurs e.g. in scientific discussion. Some inconsistencies render any further discussion impossible. Others, however, are less than fatal. As a matter of fact, people have to disagree over some issue to make a discussion start.

Now it seems that the *ex contradictione quodlibet* principle leads from any formal description of inconsistent opinions to the truth of any possible opinion, i.e. to overfilledness. Yet such a conclusion would be too hasty. The *ex falso quodlibet* principle does not match what is going on in a standard scientific, whence perfectly rational discussion. If two opinions contradict each other, then at least one must be wrong. So some participants hold false convictions at some moment of time. But nobody ever assumes that therefore everything is true.

Consequently, one should dismiss the *ex falso quodlibet* principle as inappropriate. However, not so the *ex contradictione quodlibet*, which is taken to be the very keystone of rationality in our cultural tradition. To some extent, Jaśkowski's approach is based on this problem. I do not think it is necessary to outline the technical details of the formal construction here².

Besides the interesting formal properties of the approach, it is underpinned with a well elaborated philosophical motivation, called "parainconsistency". The main idea is to handle any conflicting information in scientific theories, or databases, or belief sets, or whatever, as merely apparent inconsistencies, as *parainconsistencies*. This does not mean, however, that we ignore or avoid problems by using linguistic tricks. Inconsistencies are interesting and often highly important items in the development of the sciences and should be treated as such. Yet within this approach one is not forced to admit any inherently contradictory nature of reality. That should embank the flood of postmodern as-you-want-ities and protect that kind of logical investigation from philosophical suspicions. Any post-modern talk about fading differences between truth and falsehood sounds more than strange from a logical position. Logic lives on the difference between truth and falsehood in the very same way as ethics depends on the difference between good and evil. Naturally, parainconsistency does not claim that falsehood and the truth reconcile in the realm of logic. Quite the contrary: the aim is to obtain a precise, but more sophisticated concept of inconsistency. Not every single inconsistency amounts to a plain contradiction. However, under some circumstances it may be reasonable to treat it as a contradiction (since by

² According information can be found e.g. in Urchs 1994.

that we may gain a simplified formalism or be able to work within a more transparent calculus.) But even then modern logic has the tools to prevent such an inconsistent calculus from explosion.

The last example I will mention here is also in a sense connected with the name of Stanisław Jaśkowski.

4. Causal Logic

Causal analysis may well be considered to be one of the most sophisticated topics in analytic philosophy. And it is one of the most interesting areas for the application of logic as well. Yet at the same time, the issue contains a few extraordinarily complicated aspects. Nancy Cartwright once put it this way: “The trouble with causality is not that we cannot see it, but rather that we cannot understand it.” That does not mean, of course, that any important philosopher might hesitate to write a large monograph on causality. Actually, there is no shortage in such volumina — although they rarely meet the standards of transparency and precision required by further logical processing.

One may doubt whether there is such a thing as causality in general. Causal nexus in thermodynamics differs considerably from causal relations in history and both might be far from what a biologist would be ready to call a causal connection. There seems to be a large variety of such dependencies, rather than a single one.

What to do in such a situation? Perhaps, something else? But there is obviously a need for metamathematical counterparts of causal relations. For instance, robotics is desperately looking for reasonable causal simulators.

There are three or four main ways to approach the issue:

1. on the basis of more or less clearly formulated intuitions one defines formal connectives and names them “causal junctors”, “causal connectives”, and the like; subsequently however, these nominal definitions should be justified (by showing their adequacy to notions of causality functioning in real language), since otherwise they are not justified at all. This approach may be labelled “Collegium Logicum”. It leads to elegant formal calculi, however — as mentioned above — with minimal practical relevance;
2. beginning with causal terminology as used in some specified realm of natural language (say, in a given empirical science) one constructs a large manifold of connectives in which formal properties vary to some extent. Thus one may hope to cover all possible intuitions of the causal

notions that are used in the relevant context. So, the constructed connectives shall contain all the metamathematical properties which make them appropriate formalizations of causal nexus in the considered realm. It remains to figure out the “right” formalisation subsequently. One may refer to this as the “crazy tailor”³. The advantages of such an approach are the same as in the above case, its obvious disadvantage is the impossibility of finding out the proper formalisation by logical means alone;

3. constructing an axiomatic system as an implicit definition of causal connectives. This could be called “axiomatics”. It is a very intuitive method — unfortunately, there is almost no material for making axioms (to that purpose one would wish to have generally accepted positive features of the causal nexus — but one would hardly find enough of them);
4. starting from well-founded ontological assumptions concerning the real world, one designs all possible (i.e. consistent with the ontological settings about the structure of the world) kinds of causal connections and distinguishes then the cases of practical relevance, i.e. the kinds of causal nexus to be found in the real world. This approach could be named “Ontologic”. Again, it yields elegant calculi, from a philosophical point of view. However, the method is far from being effective.

All of them are closely related to causal logic, and in each case it was prominent Polish philosophers who originated the investigations to a considerable extent. Thus, Łukasiewicz was (besides Hugh McColl) the first who wrote on causal logic. Secondly, Stanisław Jaśkowski published his pioneering work “On the Modal and Causal Functions in Symbolic Logic” only after WW II. Third, one of the very first axiomatic systems in causal logic was given by Henryk Greniewski in Greniewski 1925. And last but not least, Roman Ingarden elaborated the formal-ontological approach to causal analysis with remarkable complexity and detail.

The contributions of these prominent thinkers are not exceptional efforts on an issue which was exotic in contemporary Polish philosophy. Quite the

³ Because of Stanisław Lem’s witty essay from *Summa technologiæ* where there is a tailor, who makes clothes. He does not know anything about people, animals, or the world. He doesn’t care about these things — he makes clothes. They look quite unusual: small or large, elastic or stiff, having no holes at all or any number of tubes, which he calls “sleeves” and “pants”, consisting of various pieces. If he finishes a dress, he takes it into a large storehouse. In that place one can find suits that fit a man or a horse or a tree, clothes for dinosaurs, unicorns, mermaids or beings unknown to anybody on earth. Every man must confess that the work of this tailor is sheer madness.

contrary, investigations in causal analysis, undertaken by formal means, belonged to the very core of analytic philosophy in Poland at the end of the last century. It is sufficient to take a look into the leading Polish philosophical journal of that time, i.e. *Przegląd Filozoficzny*. Articles which overview the causal theories of Spinoza, Kant or Mill are scattered through the journal from its very beginning, and in 1906 a special issue was devoted to the problem of causality. It contains the four prize-winning papers of a competition on causal analysis, in which Łukasiewicz's "Analysis and Construction of the Notion of Cause" was judged best, though the jury asked for a "proof of reality" of the defined notion of cause. A later competition, in 1910, ended without awarding a first prize because, as the jury put it, "there is insufficient treatment of the notion of causality". The next competition was immediately announced under the theme "Causality and Functional Connection". In 1925 (more than a quarter of a century before Burks) one of the rare investigations in axiomatic causal logic was published: Greniewski's paper "An attempt at a deductive causal theory" aimed at a direct characterization of causal nexus, without the usual "semantic detour". Though he fell victim to the usual reduction of *propter hoc* to *post hoc* (all causal concepts in his theory are definable by the relation "... is not later than ..."), his proposition is still very remarkable.

In the early fifties one of the most promising attempts at causal logic was published by Stanisław Jaśkowski. Remarkably enough, he carried out his construction with the explicit intention of applying it (namely in jurisprudence) and at the same time his metamathematical approach was meant to formalize Ingarden's conception of a cause, as sketched in Ingarden 1964 and fully worked out in Ingarden 1974. Jaśkowski's construction was further investigated by August Pieczkowski in the late sixties (see Pieczkowski 1968, 1974) and widely generalized in my present work (cf. e.g. Urchs 1994). This approach, though technically rather complex, seems to be — due to its solid philosophical foundation and unusual flexibility — one of the most interesting proposals in causal logic.

Nowadays other lines of investigation are followed rather sporadically. In 1974 Trybulec published a paper on causal operators based on mere temporal succession. Powerful semantic tools for causal logic have been developed by Wolniewicz in his Wolniewicz 1985. Still more investigations set out from Łukasiewicz's "On determinism" (e.g. Trzęsicki 1988, Urchs 1992 and the ongoing research of Tomasz Placek). Finally, there are sporadic attempts to provide an axiomatization. All of these suffer from either weakness or implausibility. That means either that there are very few axioms

only (which makes the resulting axiomatic calculus poorly equipped with interesting theorems) or we have more, but less intuitive axioms. A recent example is Kiczuk 1995, 138–157. In my opinion, Kiczuk’s axiomatization in terms of temporal succession, change and some transitive, asymmetrical, “change transferring”, temporally conservative, adjunctive causal connective with everlasting effects is definitely at odds with intuition.

5. Knowledge Dynamics

This last example is what the logician would call a counter-example. In the seventies, a group of young logicians connected with the Logic Department of the Polish Academy of Sciences was working on general consequence operations. As it turned out later, this work was one of the pillars supporting the large construction of knowledge change.⁴ Unfortunately, nobody in Poland saw the great future of these investigations, or at least nobody pushed them

⁴ David Makinson in personal communication:

“If I remember correctly, when Peter Gärdenfors began working on belief revision, he thought of the background consequence as THE classical RELATION \vdash , whereas when I began working on the same subject, I thought of the background consequence as ANY supraclassical OPERATION Cn satisfying a couple of conditions. In doing so, [...] I was indeed influenced by the Polish tradition of studying and using consequence operations, having got used to that tradition during the 70’s when reading and corresponding with Wójcicki’s group on topics like structural completeness and writing the little paper that you kindly mention [Makinson 1976]. From the beginning of my work on belief revision, I was collaborating with Carlos Alchourron, and he accepted this approach. Although Peter’s gestalt was rather different, he went along with it (and also my general notation) in our joint AGM 1985 JSL paper on partial meet contraction and revision.

In the distinct but closely related area of nonmonotonic reasoning, I also worked primarily in terms of consequence operations, and this conception is reflected in the overview paper “General patterns in nonmonotonic reasoning” in: Gabbay et al. (eds.) *Handbook of Logic in Artificial Intelligence and Logic Programming* vol. 3 Oxford 1994. On the other hand, others working in the same area, and in the subarea of preferential models for nonmonotonic inference (eg. Shoham and especially Daniel Lehmann) tended to use relations \vdash (monotonic) and \rightsquigarrow (nonmonotonic) rather than operations Cn (monotonic) and C (nonmonotonic). There is some discussion of the advantages and disadvantages of each in the overview paper mentioned.

In general, the presentation in terms of operations became dominant in the literature on belief change, while the presentation in terms of relations is currently dominant in the literature on nonmonotonic reasoning.

I would mention also one other, more intangible thing. In order to understand what is going on in the logic of nonmonotonic reasoning, one has to understand very clearly from the start that, in contrast to a long history of work in classical and non-classical propositional logics, the nonmonotonic inference relations \rightsquigarrow and their operation counter-

forward energetically enough. So nowadays this line of research cannot be counted as a Polish tradition.

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parts C are, in all interesting cases, NOT structural. (Likewise, revision and contraction operations are not structural). It was only by becoming familiar in the 70's with Polish investigations on structural completeness, that I was prepared to make the formally small, but psychologically immense, transition to considering non-structural consequence relations/operations."

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