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Jerzy Perzanowski

LOCATIVE ONTOLOGY

Parts I – III

To the memory of Władysław Kania
(1924–1992)

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CONTENTS

PART I. BACKGROUND

1. INTRODUCTION

1.1 Being is said and interpreted in various ways. These are studied in ontology. Ontology is concerned with both *particular beings* of several sorts as well as with *the being* and *Being* itself — respectively beings' *collection* and *unification into one*[1].

1.2 Ontology is the theory of *what there is*, and of *why* and *how*. It can be either *descriptive* (phenomenological), putting emphasis on the first component of the ontological question "what there is", or *theoretical* (speculative), trying to outline a logical view of its universe — the ontological universe.

Following Leibniz this space is here understood as the space of all possibilities, for to the second part of the ontological question "why and how?" we are looking for an answer of the form "x *is* because x *is possible* and, in addition, enjoys additional specific conditions".

1.3 Three closely related ontological notions are basic: the notion of *a being*, the notion of *the being* and the notion of *Being.* These are, to be sure, obscure and complex, covering under three expressions a rich variety of connected ideas.

They can be approached in at least three ways[2] *connectional* or *qualitative*, and through what we shall call *verb-type* or *relational* ontologies.

1.4 The possibilistic approach is determined by Leibniz's question "How (a given) is possible?". A being is defined here as any possible object. In consequence, ontology equals the general theory of possibility.

[1] Cf. [15]

[2] For an extended modern discussion of all three approaches cf. [13], [15] and [23]. The possibilistic approach is discussed also in [14], [19], [21] and [22]; the qualitative one in [16] and [17],

The qualitative, or connectional, approach develops the very traditional idea: *a being is any subject of some qualities.* The ontology here is confined to the theory of qualities, of subjects and of the connections between them.

The verb-type approach starts with an obvious observation that basic ontological notions are nominal derivatives of the verb "to be". Hence it consists in a clarification of the nominalizations of the verb "to be". Because of an influential traditional reducing of all affirmative statements to sentences of the form "S is P", verb-type-ontology is dominated by its attributive or predicative variants.

1.5 Locative ontology, which I am going to discuss here, is a variant of verb-type-ontology determined by locative uses of the verb "to be", like: *I am here. You are at home. She is in Schaan.* But also *I am in trouble* (in writing this essay), etc.

It has been observed[3] that locative uses are among the most primitive forms of the verb "to be" in Indo-European languages. On the other hand, they played a crucial role in the development of certain basic ontological concepts of ancient Greek philosophy[4].

Notice next, that it is very unnatural to impose upon locative sentences the canonical form "S is P". To this end people usually claim that "in Schaan" is a predicate or paraphrase "She is in Schaan" into "She is in a state of being in Schaan". Both approaches are transparently artificial.

In spite of bearing marks of outstanding ontological importance, locative sentences have been almost never discussed in the literature of ontology.

1.6 The present essay intends to cover this gap. It forms a first, still very preliminary, step into *combination ontology.*

Namely, *combination* can be treated as *location* plus *connection.* In turn, *combination metaphysics* can be made by experience with the idea that *existence* is *combination*, i.e. *location* plus *connection*, plus *condensation* plus *stabilization* (plus, perhaps, something else). In short:

combination = **location** + **connection**
existence = **combination** + **condensation** + **stabilization** + ...

1.7 The idea which I am going to develop here is as follows: *x is located in y* if and only if *each of its parts is in y.*

[3] Cf. Brückner [2]
[4] Cf. Kahn [5]

1.8 The paper is organized as follows: I start with a general and brief overview of verb-type-ontologies, stressing the importance of the locative one. Next, three main relavant formal theories — of preorders, of mereologies as well as Leśniewski's Ontology — are presented. They are shown to be inadequate to formalise location.

In this survey a special emphasis is put on premereologies intermediate between classical mereologies and preorders. Premereology seems to be very useful in the field of ontology and metaphysics as the first, purely logical, approximation of the idea of condensation, i.e. the internal strength of unifying connections.

Next, I will pass to a discussion of locative ontologies, introducing them as a generalization of preorders, which fill in certain gaps occurring in both mathematical and philosophical approaches to orders. The bulk of locative ontology is presented in the Parts II and III, where locative orders are introduced and related to more familiar structures outlined previously. At the end, the philosophical content of locative ontology is presented and, finally, several cases of location in some important domais are pointed out.

1.9 The present paper is an essay in mathematical philosophy[5]: its problems are philosophical, its procedure is mathematical. In particular, in the exposition of feel free to behave like in mathematical study.

The work has two aims: a philosophical one — to clarify one of the most important variants of verb-type-ontology, and a mathematical one — to enlarge the body of commonly known theories of orders.

2. VERB-TYPE-ONTOLOGIES

2.1 The best way, I think, to introduce a variety of verb ontologies is to introduce a bit of grammar. Indeed, verbs, including the verb "to be", play a crucial role in generating verb ontologies.

2.2 Let's start with a list[6] of the kernel affirmative English sentences.

Hereafter, N denotes nouns, A — adjectives, V — verbs, P — prepositions, D — phrases of description, qualification or classification, M —

[5] Recall Leibniz-Russell-Łukasiewicz's program of mathematical philosophy.

[6] I have here followed, *mutatis mutandis*, Z. Harris and C. Kahn's list, cf. Kahn's [5], relying on my [12].

mereological phrases, L — locative phrases, S sentences. Star (or copula) ∗ is reserved to the verb "to be".

V-SENTENCES

with verbs not reducible to the verb "to be"

They are state or processual sentences of two sorts:

Intransitive

NV	John sleeps.

Transitive

NVN	John loves Mary.
NVPN	John is looking at Mary.

BV-SENTENCES

specific for English
expressing the presence and continuity of processes

N∗D(V)	John is sleeping.
N∗D(VN)	John is loving Mary.
N∗D(VPN)	John is looking at Mary.

B-SENTENCES

with "to be" as their verb

They come in so rich a variety that some people believe that "is" in general is not a proper verb but only a formal copula.

NOMINAL

D-type

N∗A	John is old.
N∗N	John is a man.
N∗AN	John is a good man.

M-type

N∗MN	A roof is a part of a house.

Possessive

N∗D(N)	This book is mine.

ADVERBIAL

$N{*}D_1(VD_2)$	John is sleeping silently.

PASSIVE

N∗D	John is loved.

LOCATIVE

N∗PN	John is at home.
N∗L	John is here.
N∗L(PN)	John is with Mary.

IDENTITY

N∗N	The Evening Star is the Morning Star.
N(S)∗N(S)	My love is my life.
N∗S	John's fear is: I have cancer.
S∗S	That John has AIDS is: John has illness more dangerous than cancer.

EXISTENTIAL

N∗	John is.

2.3 There is a clear correspondence[7] between the above spectrum of kernel affirmative statements and the main kinds of ontologies.

B- and BV-sentences correspond in general to change and process ontologies.

For B-sentences the story is much more complex. Both *being* and the verb "to be" are indeed said and interpreted in various ways. Nominal sentences of D-type correspond to object-property ontology as well as to ontologies of attributes, predicates and multiplicities. M-type sentences generate mereologies, locative statements generate locative and combination ontologies. Adverbial and possessive sentences are connected with ontology of states, whereas existential statements with existential ontology[8].

Which specific ontology, if any, is connected with identity statements remains unclear.

2.4 In sum, the spectrum of ontologies coincides with the variety of kernel sentences. It seems that each language, when developed sufficiently, enables us to express ontological ideas in a very economical way: different means for different pictures of the world.

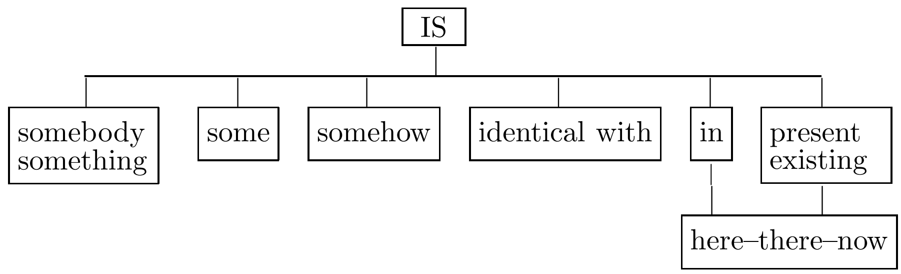
2.5 In spite of their differences, the many uses of the verb "to be" all have something in common.

"Is" is not ambiguous. Also, it is not an empty, purely formal, copula.

It is rather a *very general* verb, open for particularization and variation.

2.6 Its mechanism can be made fairly clear by observing that the kernel sentences are, in fact, generated by the main questions we use to express our curiosity: What? Who? Which? When? Where? How? Why? In which way? etc.

Classifying information we obtain the following variants of the verb "is":



[7] For details cf. [13] and [23].

[8] In particular, with so-called existential thomism of Gilson. Cf. Krąpiec [6].

The last two variants clearly generate mereological, collective and locative perspectives on the world.

2.7 It is useful to compare the background of the well established mereological formalisation of the verb "to be" with the locative approach, which we are going to study here.

The common factor of both variants lies in the question-answer pair ***where — in***, which is common to them. On the other hand, their difference can be made clear by connecting the pair ***who/what — somebody/something (a part of)*** with the mereological use of "is", whereas the pair ***where/in which way — present-existing (by or through location)*** is connected with the locative variant of "is".

Both variants have thereby something in common, but they also differ in a quite sharp way. As it will be made clear later on, locative ontology, although close to mereology, is not reducible to it.

3. VERB-TYPE-ONTOLOGICS

3.1 I shall use the term "ontologic" to refer to the formal counterpart of ontology.

Usually, a given ontology generates a bundle of connected ontologics, its fairly complete formal developments.

3.2 Verb ontologics can be divided into two big families: *transformation* or *process ontologics*, generated by V-sentences, which formalize change ontologies, and *be-logics,* generated by B-sentences and formalizing *be*-ing ontologies.

By the nature of the verb "to be", the most general be-logic is the general theory of relations[9]

3.3 The first step towards formalization is introduction of a suitable notation.

In principle, the symbol E will be used instead of the verb "is" in its most general reading: *is related to.* I.e., for arbitrary items x and y, and for appropriate variant of "is", the expression "$E(x, y)$" means "x is y". Following the syntax of natural languages we shall usually write "xEy"

[9] Like Schröder and Tarski's theory, cf. [24] and [28]. Notice that in the 1980s the general, set-theoretical theory of relations was revived under the name *theory of generalized quantifiers.* cf. van Benthem [1] and Westerståhl [29].

instead of "$E(x, y)$". Notice, however, that in the most general part of the present paper the symbol E denotes, in fact, an arbitrarily chosen, but fixed binary relation.

3.4 Two types of be-logics should in general be distinguished: *standard* or *elementary*, and *non-standard* or *propositional* ones. Both assume classical quantificational logic. The difference is syntactical.

In the elementary approach we treat E as a distinguished two-place predicate letter, which builds up atomic formulas only. These formulas, in turn, are used to build up complex formulas by means of the standard classical logical connectives: negation – $\neg$, conjunction – $\wedge$, disjunction – $\vee$, implication – $\rightarrow$, equivalence – $\leftrightarrow$, and minor and major quantifiers: $\exists$ and $\forall$.

Therefore, in the standard case complex formulas are build up only by means of logical connectives. The primitives of a theory, its specific symbols, occur only in terms, if any, and in atomic formulas. For example, the expression $E(x, y)$ is allowed, but $E(E(x, x), z)$ is not. This might be considered an unjust limitation, for the statement "That x is y, is z" looks quite reasonable.

On the other hand, in the non-standard, propositional approach by means of E we can build both atomic and complex formulas. There the expression $E(E(x, y), z)$ is well-formed.

3.5 People usually follow the standard approach, in which belogics are simply elementary theories of *specific* binary relations. By specification we obtain inter alia:

Set-theoretical ontologics; here E equals $\in$ (to be member of), i.e., $xEy := x \in y$.

Inclusion, or ***Boolean*** algebraic ontologics, by considering "to be included in". Here E equals $\subset$, i.e., $xEy := x \subset y$.

Mereological ontologics, or simply ***mereologies***, by considering "to be a part of". Here E equals $<$, i.e., $xEy := tx < y$.

Predication ontologics, or predicate calculi by considering "to be predicated by". Here E equals predication, i.e., $xEy := y(x)$.

Attribution ontologics or ***property calculi*** by considering "being an attribute of". Here E equals attribution, i.e., $xEy := y[x]$.

Nominal identity ontologics, by considering "to be identical with". Here E equals identity, i.e., $xEy := (x = y)$.

3.6 By comparison of the above list of the main kinds of belogics occurring in the literature with the previous list of the main be-ontologies we note several gaps in the first list.

The most urgent is the lack of suitable locative ontologics. In what follows, then, I shall try to fill this gap.

3.7 Finally, let's outline[10] very briefly the *general* scheme defining in *each* case, i.e. for any arbitrarily chosen but fixed relation E its basic ontological notions:

x is ***E-being*** := For some y, x is y : $\exists y\, xEy$. Either more generally
x is ***E-being*** := For some y, xEy or conversely for some y, yEx:
$(\exists y\, xEy) \vee (\exists y\, yEx)$

A being is everything which is something or which is of something.

To define *the being* we need a collecting operator: $\{x :\ A(x)\}$ denotes the collection of all objects satisfying the condition A. Now,

the E-being := $\{x :\ \exists y\, xEy\}$ or more generally
$\{x :\ (\exists y\, xEy) \vee (\exists y\, yEx)\}$.

The being is the collection of all beings.

To define *Being* we need a unifying operator: $[x :\ A(x)]$ denotes the unity of all objects satisfying the condition A. Now,

E-Being := $[x :\ A(x)]$ or more generally $[x :\ (\exists y\, xEy) \vee (\exists y\, yEx)]$.

Being is the unity of all beings.

3.8 The basic idea is quite natural: beings are items which *are*, i.e., objects of *is* — connection.

Observe that in this way we connect verb ontologies with qualitative ones.

3.9 In conclusion, investigating particular be-logics we indeed investigate suitable ontologies.

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[10] For details cf. [15].

4. RUDIMENTARY DEFINITIONS

4.1 Hereafter, I am following the standard, elementary approach. I.e., I am working in a suitable version of the classical predicate calculus with identity[11], with *one* non-logical primitive — a binary predicate letter E.

In general E should be treated as denoting any arbitrarily chosen but fixed binary relation.

4.2 The standard mathematical notions are used without any ceremony.

In particular the *converse* of E, E^{-1}, is defined by the condition $xE^{-1}y := yEx$, whereas the *superposition* of two binary relations E and E', $E \circ E'$, is defined by:

$$xE \circ E'y := \exists z\ (xEz \wedge zE'y);$$

x is in the $E \circ E'$ relation to y if and only if some object z mediates between them: xEz and $zE'y$.

4.3 Recall that the following kinds of binary relations are of primary mathematical interest: the transitive, symmetric, antisymmetric and reflexive ones. By their combination we obtain the most important mathematical orders: equivalences, preorders and partial orders.

For readers's convenience I shall repeat below their usual definitions.

E is *transitive*, if it fulfils the following transitivity condition:

T $$\forall x \forall y \forall z\ (xEy \wedge yEz \rightarrow xEz)$$

For any x, y and z, if x is related to y and y to z then x is related to z as well.

NB. It is a well-established custom, which I will also follow, to omit prefixed general quantifiers.

E is *symmetric*, if it fulfils the following symmetricity condition:

S $$xEy \rightarrow yEx$$

If x is related to y, then also conversely, y is related to x.

Usually two types of opposite condition are introduced:

[11] Cf. any logical textbook, for example Mendelson [11].

Asymmetry

AS $$xEy \wedge yEx \rightarrow x = y$$

Anti-symmetry

A − S $$xEy \rightarrow \neg(yEx)$$

E is anti-symmetrical, if it is not reversible.

E is *reflexive*, if it fulfils the following reflexivity condition

R $$xEx$$

Anything is related to itself.

Recalling our ontological interest we can note that for "is" of identity reflexivity becomes the *Identity Principle*: *Everything is self-identical.*

4.4 Let me point out also that usually relations are considered only in restriction to a given or presupposed class U.

4.5 The smallest reflexive relation on a given set X is named its *diagonal* and is denoted by Δ_X.

$$\Delta_X := \{\langle x, x\rangle : \ x \in X\}$$

4.6 By combination of the above conditions we obtain:

Equivalence relations — these are just the reflexive, transitive and symmetric relations:

EQ $$\mathbf{R} \wedge \mathbf{T} \wedge \mathbf{S}$$

Preorders — these are just the reflexive and transitive relations:

PO $$\mathbf{R} \wedge \mathbf{T}$$

Partial orders — these are just the transitive and asymmetric relations:

POR $$\mathbf{T} \wedge \mathbf{AS}$$

Strict partial orders — these are just the transitive and anti-symmetric relations:

SPOR $$\mathbf{T} \wedge \mathbf{A{-}S}$$

4.7 One general assumption: E is hereafter, if not additionally specified, an arbitrarily chosen binary relation on a fixed universe U. Usually, the universe parameter is omitted. For example, the diagonal of a given U is denoted simply by Δ instead of Δ_U.

4.8 Relations are compared by means of inclusion, i.e., $R \leqslant S$ *iff the extension of R is included into the extension of S.*

4.9 Recall the following nice characterization of the above types of relations (cf. Cohn [3]):

(1)	E is transitive	iff $E \circ E \leqslant E$, or $E^2 \leqslant E$
	E is symmetric	iff $E \leqslant E^{-1}$, or $E = E^{-1}$
	E is asymmetric	iff $E \cap E^{-1} \leqslant \Delta$
	E is antisymmetric	iff $E \cap E^{-1} = \emptyset$
	E is reflexive	iff $\Delta \leqslant E$
	E is an equivalence	iff $\Delta \cup E^2 \leqslant E = E^{-1}$
	E is a preorder	iff $\Delta \cup E^2 \leqslant E$
	E is a partial order	iff $E^2 \leqslant E$ and $E \cap E^{-1} \leqslant \Delta$
	E is a strict partial order	iff $E^2 \leqslant E$ and $E \cap E^{-1} = \emptyset$

4.10 The above theorem suggests that the most natural kinds of relations are characterizable by suitable conditions of comparison. Indeed, reflexivity of E means that the diagonal relation is weaker than it: $\Delta \leqslant E$; transitivity of E that $E^2 \leqslant E$, etc.

The most natural relational axioms are therefore conditions of comparison.

4.11 Hereafter the following two conventions are in use:

A For given relations Q and R their suitable *comparison conditions* are as follows:

QR $\qquad Q \leqslant R$

QR $\qquad Q = R$

B For any relational condition ϕ, $\boldsymbol{\phi}\mathbf{S}$ denotes the class of its models:

$$\boldsymbol{\phi}\mathbf{S} := \{\langle U, E\rangle : \ \langle U, E\rangle \ \text{ satisfies } \phi\}$$

In particular, **TS** denotes the class of all transitive structures, **RS** — the class of all reflexive structures, **POS** — the class of all preorders, etc.

4.12 Let me mention also *filters* and *ideals*. These are two extremely useful order-concepts.

For a given x, by a relation filter $[x)$ generated by x we mean the class of all objects related to x, whereas a suitable relation ideal $(x]$ is the family of all objects to which x is related. In symbols:

$$\begin{array}{lll} [x) & := & \{y : \ xEy\} \\ (x] & := & \{y : \ yEx\} \end{array}$$

Observe that x belongs to both sets iff E is reflexive. Therefore, for reflexive E it is useful to distinguish the body of a filter from the filter itself:

$$[x] := \{y : \ xEy \wedge x \neq y\}$$

and likewise the body of the ideal from the ideal itself:

$$(x) := \{y : \ yEx \wedge x \neq y\}.$$

To explain our terminology recall Leibnizian idea of the body of a whole as collection of all items subordinated to whole's central element. Expressing this idea in order-terms we see that the body of an ideal $(x]$, i.e. (x), plays indeed the role of x-body.

4.13 Finally notice that the most reasonable mathematical candidates for the formalization of the "is in" of locative ontology are preorders or partial orders. You will see, however, that our well-motivated candidate is different, though quite similar.

4.14 In order to introduce locative ontology in a natural way, I will start by collecting certain basic observations concerning well-established and commonly known mathematical and mereological structures.

First I will discuss preorders, next mereologies, passing finally to the proper topic of the paper.

5. PREORDERS

5.1 Observe first that the condition

$$xPy := \forall z\ (zEx \rightarrow zEy), \text{ i.e., } xPy := (x] \subseteq (y]$$

defines, for any E, its conjugate preorder:

(2) *P is a preorder, i.e., a reflexive and transitive relation*[12].

Proof. We obtain this immediately, after suitable substitutions, by the identity and transitivity laws for classical implication.

Indeed, xPy iff $\forall z\ (zEx \to zEy)$, whereas $xPy \wedge yPu$ iff $\forall z\ (zEx \to zEy) \wedge \forall z\ (zEy \to zEu)$ iff $\forall z\ ((zEx \to zEy) \wedge (zEy \to zEu))$, hence $\forall z\ (zEx \to zEu)$, i.e., xPu.

5.2 What is the right way of reading P?

For the ontological understanding of E (E equals *is*), xPy means that *everything which is x is also y*. This sounds like quite a good expression of the ontological presupposition of the *part-whole* relation. For this reason, xPy is read: x is a part of y.

Notice that in the next chapter stronger mereological part-whole relations will be introduced.

5.3 Let us now assemble a list of *all* binary relations defined by means of generally closed implications connecting formulas from the list: zEx, zEy, xEz, yEz in all possible ways.

There are exactly eight such possibilities. Four of them define converses of the other four. Hence only four definitions are interesting.

The above definition of P is one. The remaining three are the following:
$xCy := \forall z\ (yEz \to xEz)$, i.e. $[y) \subseteq [x)$, which is read: *x is covered by y*
$xHy := \forall z\ (zEx \to yEz)$, i.e. $(x] \subseteq [y)$, which is read: *x houses y*
$xDy := \forall z\ (yEz \to zEx)$, i.e. $[y) \subseteq (x]$, which is read: *x dominates y*

As a matter of fact, the four relations indicated above correspond to all cases of inclusion between x and y — filters and ideals: $[x)$, $(x]$ compared with $[y)$, $(y]$.

As regards reading, the above proposals are intended not to be *ad hoc*. For example, to justify covering think of E as enveloping, i.e., yEz means: z is an envelope of y or y is enveloped by z. Now, y covers x, if each envelope of y is at the same time envelope of x. Isn't it?

5.4 Notice that C, like P, is in general a preorder:

[12] Strictly speaking we should parametrize P by writing P_E. Remember, however, that by our previous convention E is fixed, hence we can omit it. The proof given in the text is very elemntary. I am giving it here with all details as paradigmatic case to be free to omit such elemntary arguments in the future.

(3) *C is a preorder relation.*

5.5 As a matter of fact, both P and C are characteristic preorder relations. Namely, by comparison of our primitive relation E with P, or with C, we can characterize preorders.

To this end observe first

(4) *i) E is reflexive iff $P \leqslant E$ iff $C \leqslant E$*
ii) E is transitive iff $E \leqslant P$ iff $E \leqslant C$

Proof. I will check only two of the four claimed equivalences, the remaining two leaving to you.

Ad i) If E is reflexive, i.e., xEx then because xPy implies $xEx \to xEy$, we obtain that xEy. Hence $P \leqslant E$.

Conversely, if $\forall x \forall y\ (xPy \to xEy)$, then $xPx \to xEx$. But, by (2), P is reflexive. Hence xEx, i.e., E is reflexive as well.

Ad ii) Let $E \leqslant C$, i.e., $\forall x \forall y\ (xEy \to xCy)$. Hence $\forall x \forall y\ (xEy \to \forall z\ (yEz \to xEz))$. This, by classical logic, is equivalent to $\forall x \forall y \forall z\ (xEy \to (yEz \to xEz))$, which is the transitivity condition for E.

The reverse implication can be checked simply by reversing the above reasoning.

As immediate corollary we obtain the following characterization of preorders

(5) *E is a preorder relation iff $E = P$ iff $E = C$ iff $P = E = C$.*

5.6 There is an important philosophical problem to find the *right* axiomatization of the part-whole relation.

The problem clearly has at least two components: one purely formal — concerning the relation *to be a part of*, and the other, more essential, though still formal — concerning *wholeness.*

Therefore, if we take preorders to be the first and very general approximation of the relation *to be a part of*, we can claim that the formula[13]

(**PE**) $$P = E$$

saying: *to be* is *to be a part of*, by its generality, formalizes (onto)logical mereology. In such a case, (onto)logical mereologies are simply preorders.

[13] Similarly for equivalent axioms: (**EC**) $E = C$, and (**PEC**) $P = E = C$.

5.7 Equivalence relations are preorders of strong form. Notice that they can be characterized in a way similar to one given in (5).

First recall that by (1), E is symmetric iff $E = E^{-1}$. Combining this with (5) we obtain the following characterization of equivalence relations:

(6) *E is an equivalence relation iff $E^{-1} = P$ iff $E^{-1} = C$ iff $P = E^{-1} = C$.*

Proof. First notice that E is reflexive, or transitive, or symmetric, or an equivalence relation iff its converse E^{-1} fulfils the respective condition.

I will check only the first equivalence in our claim, for the remaining two are similar.

Let E be a relation of equivalence. By its symmetricity $E = E^{-1}$, whereas by its being preorder $E = P$. Hence $E^{-1} = P$.

Conversely, let $E^{-1} = P$. By (5), E^{-1} is a preorder relation, hence by the first observation of the proof, E is a preorder as well. Repeating (5) we obtain $E = P$. Therefore $E = E^{-1}$, hence E is symmetric. In conclusion, E is a symmetric preorder, i.e., an equivalence relation.

5.8 To conclude, relations P and C are indeed characteristic preorders.

As regards the remaining two relations H and D, they are, in a sense, connected with symmetricity.

First of all, observe that they connect symmetricity with reflexivity:

(7) *i) E is symmetric iff H is reflexive iff D is reflexive*
ii) E is symmetric iff $P = H$ iff $C = D$

Proof. The case (i) and the right-hand implication in (ii) follow immediately from the definitions.

For the left-hand implication in (ii), assume that $P = H$. Hence H is a preorder, for P is. Therefore H is reflexive. By (i), E is symmetric, as required.

Next observe

(8) *If E is symmetric, i.e., if H and D are reflexive, then both H and D are transitive, hence preorders.*

5.9 Combining the above results with the previous ones we finally reach

(9) *E is an equivalence relation iff $E = P = C = H = D$.*

Therefore, in the case of equivalence relations all 10 relations involved: E, P, C, H, D and their converse relations, coincide.

Equivalence relations are very regular indeed!

5.10 Query. It is easy to see that four-element equality: $P = C = H = D$ doesn't entail that E is equivalence.

It characterizes thereby some broader class of symmetric relations. Which one?

5.11 Using the four relations: P, C, H and D, and in particular the first two, we can obtain quite aesthetic characterizations of the standard mathematical order-relations.

Notice, however, a remarkable gap. Until now we didn't compare P and C in an immediate way, but only by means of the basic relation E. It will become clear shortly that such immediate comparison means location.

5.12 Final remarks concerning partial orders. First of all, *partial orders and preorders overlap.* As a matter of fact we should distinguish reflexive partial orders, i.e. asymmetric preorders, and irreflexive ones, i.e. partial orders which are not preorders.

Interrelations between relevant classes of relations can be presented as follows:

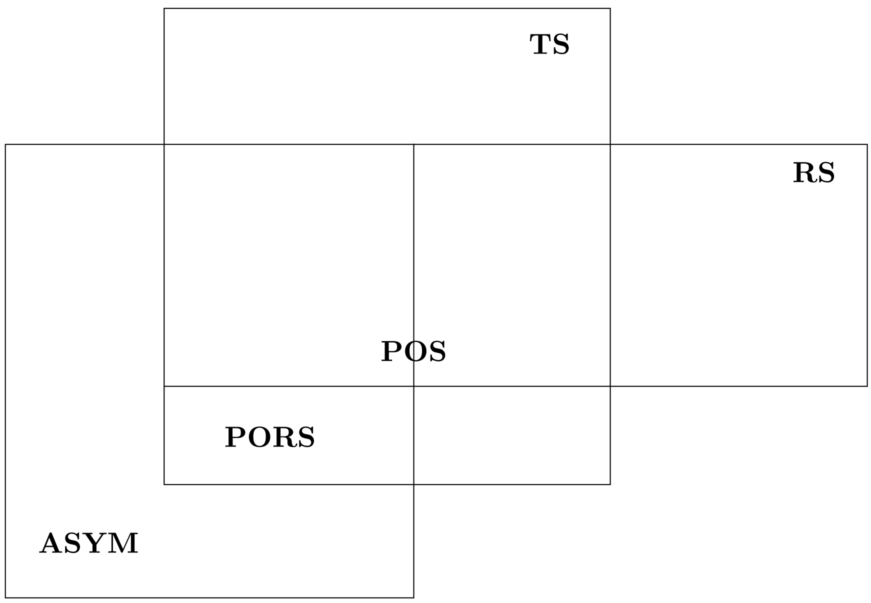


Fig. 1.

5.13 The basic fact about partial orders of the first type, i.e. asymmetric preorders, is that they are *extensional*, i.e., fulfil the extensionality condition:

Ext $$x = y \leftrightarrow \forall z\ (zEx \leftrightarrow zEy)$$

In fact, we can prove even more:

(10) *If E is reflexive and asymmetric, then E is extensional.*

Proof. The right-hand implication is due to the extensionality of classical logic.

Conversely, assuming the right-hand equivalence $\forall z\ (zEx \leftrightarrow zEy)$, by suitable particularization and using logic, we obtain $xEx \wedge yEy \leftrightarrow xEy \wedge yEx$. Hence, by reflexivity of E, $xEy \wedge yEx$. Applying now the asymmetry condition **AS** we reach the conclusion: $x = y$.

6. PREMEREOLOGIES

6.1 Proper mereologies consider both sides involved: the relation *to be a part of* and *wholes*, trying to approach the latter by means of the former.

6.2 We start by introducing a few additional notions.

The first two should be understand as general, i.e. as not specifically mereological.

$$xOy := \exists z\ (zEx \wedge zEy), \qquad \text{i.e. } (x] \cap (y] \neq \emptyset$$

x *overlaps* with y iff some object z *is* both x and y[14].

$$xDSy := \neg(xOy)$$

x is *discrete* from y, if they two do not overlap.

Now we are ready to define the mereological order relation M — *is a* (*mereological*) *part of* — by the condition saying that *everything overlapping a part overlaps also a whole*:

$$xMy := \forall z\ (zOx \rightarrow zOy), \qquad \textit{i.e.}[(x]) \subseteq [(y])$$

[14] Recall that I am consequently using the ontological reading of E by “is”.

By definition

(11) xMy iff $\forall z\,(zDSy \rightarrow zDSx)$

For brevity's sake, the relation M will often be read as "meeting relation". I.e., xMy is read: *x meets y*.

6.3 Previously, for any given binary relation E its conjugate part-relation was defined. Is M definable after this manner?

Yes. For a given relation E its meeting relation M is equal to the part-relation P taken, however, for overlapping O instead of the primitive relation E. I.e., using parameters we can write: $M_E = P_O$, or more exactly: $M_E = P_{O_E}$.

A question arises: Does $M = P$ hold in general?

Later on, cf. (15), you will see that the answer is: No, it isn't.

Mereological Condensation

6.4 The crucial mereological axiom, common for standard[15] mereological systems including suitable calculi of individuals, is the following *axiom of mereological condensation*:

ME $E = M$, i.e., $\forall x \forall y\,(xEy \leftrightarrow xMy)$
To be means *to stand to something in the mereological meeting relation.*

The axiom **ME** defines the class of *premereologies*, a realm intermediate between proper mereologies[16] and preorders.

6.5 Similarly to P and C, and for similar reasons

(12) *M is a preorder.*

Hence, if E is premereological, i.e. if it satisfies **ME**, then E is a preorder relation as well. Therefore

(13) *All premereologies are preorders.*

We may ask now, which preorders are premereological?

[15] For references a reader is addressed to P. Simons' treatise [25] or to M. Libardi's survey [10].

[16] I.e. classical extensional mereology, some of its subsystems and Leśniewski's Mereology [7], [8] and [9]. Cf. Simons [25], Libardi [10].

It is easy to check that not any preorder is such. We will see that in fact premereologies show some similarity to Boolean structures, whereas proper mereologies simply imitate them.

6.6 By (5) and (12)

(14) *If E is premereological, i.e. if it satisfies* **ME**, *then* $M = E = P$

Therefore, whereas $E = P$ is the characteristic equation for preorders, $M = P$ seems to be characteristic for premereologies. In general it is not true. Consider, for example, the empty relation. But for preorders it is.

(15) *If E is a preorder, then E is premereological iff $M = P$.*

Proof. The right-hand implication is implicit in (14).

For the reverse implication, notice that $E = P$, for E is a preorder. By the second assumption, $M = P$, hence $E = M$, i.e., E is premereological, as required.

In conclusion, in the realm of preorders premereologies identify the (onto)logical part-relation with meeting. In this case, only parts meet wholes.

Overlapping

6.7 We are going now to collect a few elementary properties of overlapping, in particular those which compare it with the other relations under investigation.

From the definitions we immediately obtain

(16) *O is symmetric in general, and reflexive provided E is such.*

As regards comparison:

(17) *If E is reflexive then $M \leqslant O$ and $P \leqslant O$. Also*, as was stated in (4), *reflexivity of E implies $P \leqslant E$*

(18) *If E transitive, then $E \leqslant M$ and*, cf.(4), $E \leqslant P$.

Hence

(19) *If E is a preorder relation, then $P = E \leqslant M \leqslant O$.*

Therefore, taking into account (15), we see that *in the realm of preorders premereologies can be characterized by the inequality* (***MP***) $M \leqslant P$.

6.8 *Let us examine the three inequalities from* (19): $P \leqslant M$, $M \leqslant O$ *and* $P \leqslant O$.

For brevity's sake *we limit consideration to the case of preorders*, i.e., we assume that $E = P$.

First, observe that in general none of the above inequalities is reversible.

As regards $M \leqslant P$ we know that in the case of preorders it axiomatizes premereologies.

Next, note that in general

(20) $O \leqslant M$ *iff* O *is transitive.*

Hence, *in the case of preorders*

(21) $O = M$ *iff* O *is transitive.*

Notice, that the restriction to preorders is essential, for even in the case of premereologies O has not to be transitive.

6.9 Finally, let's turn to the strongest equivalence: $O \leqslant P$. I.e. $O \leqslant E$, for we work with preorders only.

In this case, by (19), $E = P = M = O$. It is easy to foresee that this collapse of all relations involved onto one relation means that it is an equivalence relation. Indeed

(22) *Let* E *be a preorder.* $O \leqslant E$ *iff* E *is an equivalence relation.*

Proof. The left-hand implication is well-known. It is, in fact, one of the key cases in *the abstraction principle* connecting equivalence relations with partitions of sets.

For the reverse implication assume that E is a preorder, and that $xOy \rightarrow xEy$, i.e., $\exists z\,(zEx \wedge zEy) \rightarrow xEy$. By classical logic, $\forall z\,(zEx \wedge zEy \rightarrow xEy)$. Putting $z := y$ we obtain: $yEx \rightarrow xEy$. Hence E is a symmetric preorder. I.e., it is an equivalence relation, as required.

6.10 To sum up: Joining (9) with the previous observation we see that in the case of equivalence relations all relations yet considered collapse: $E = P = C = H = D = M = O$.

Equivalence is indeed regular. Regularity means simplicity. But too much regularity means too much simplification, killing differences as the dessert kills forests.

6.11 Notice that in general, i.e. for any relation E, the following laws of monotonicity for overlapping hold:

(23) $zOx \wedge xPy \rightarrow zOy$ *and* $zOx \wedge xMy \rightarrow zOy$;

whereas

(24) $zOx \wedge xEy \rightarrow zOy$, *provided* E *is transitive.*

If a smaller item overlaps with some third item then the bigger item overlaps with the latter as well.

Condensation Revisited

6.12 Let us return to the discussion of the mereological axiom **ME**.

First two definitions — of *proper being (related)* and being *a proper part of*:

$$xPEy := xEy \wedge x \neq y$$

x *is properly* (related to) y iff x is (related to) y, but the two differ from each other;

$$xPPy := xPy \wedge x \neq y$$

x is *a proper part of* y iff x is a part of y, but different from it.

In the realm of preorders both notions clearly coincide.

Also, assuming **AS** we have the following: *if* x *is properly* y *then* y *is not* x, $xPEy \rightarrow (yEx)$.

Similarly, in the case of asymmetric preorders, *if* x *is a proper part of* y *then* y *is neither* x *nor a part of* x*:*

$$xPPy \rightarrow \neg(yEx) \wedge \neg(yPx).$$

6.13 Now, is the time to introduce the basic ontological opposition between *atoms* (*simples* or *elements*) and *complexes.*

For a given relation E, an object x is said to be E*-simpler than* y, if xEy. Next, x is said to be an *atom* of the relational space $\langle U, E\rangle$, what we write $A(x)$, if no *different* object is simpler than it, or, using terms from 4.12, it is without body:

$$A(x) := \neg\exists z\,(zPEx), \quad \text{or} \quad (x) = \emptyset.$$

Out of preorders atoms should carefully be distinguished from simples: x is *a simple*, if *no* object is simpler than it, or if it is an atom not simpler than itself.

x is a *complex* iff it is not an atom, or it has a body:

$$C(x) := \exists z\,(zPEx), \quad \text{or} \quad (x) \neq \emptyset.$$

Atoms, simples and complexes are, for sure, basic notions of any combination, combinatorial or constructional ontology. In spite, or perhaps because of, their obviousness they are rich and fruitful notions, which clearly deserve a very detailed study.

For such an account an interested reader is referred to [19] and [23]. In the next two sections I will employ both notions to characterize premereologies. Later on I will use them to classify locative ontologies.

6.14 We are working, as usual, with preorders. I.e., E is taken to be a preorder relation.

Observe first

(25) *If* $(x) = \emptyset$ *then for any* y, xMy *implies* xEy.

Proof. As a matter of fact we will prove this using only the reflexivity of E.

Suppose that xMy, i.e., (i) $\forall z\ (zOx \to zOy)$. But x is an atom, therefore (ii) $\forall z\ (zOx \leftrightarrow xEz)$. By reflexivity of E, xEx. Hence xOx. Therefore, by (ii), xOy, i.e. yOx. Applying again (ii) we obtain xEy, as required.

Notice that in the above claim the assumption of the reflexivity of E is essential. Observe first that a simple x meets everything: *if x is simple, then for any y, xMy.* Therefore, (25) is falsified by any frame with simples which are not related to everything.

Combinig (25) with (18) we obtain

(26) *If* x *is an atom, then for any* y: $xEy \leftrightarrow xMy$.

In other words, at the very bottom of a preorder, i.e. for its atoms, the axiom **ME** holds.

Therefore the real difference between preorders and premereologies occurs only in the realm of complexes.

6.15 Hence, let's turn to the realm of complexes.
First a trivial but useful observation:

(27) *For a transitive* E, *if* xEy *then* $(x) \subseteq (y)$ *and if, in addition,* x *is complex then also* y *is complex.*

The next claim is more essential

(28) *If* x *is complex then* $(x) \subseteq (y) \to xMy$.

Proof. Assume that $(x) \neq \emptyset$, $(x) \subseteq (y)$ and that for arbitrary but fixed z, zOx. Hence for some u, uEz and uEx.

If $u = x$, then xEz. Take $\underline{u} \in (x)$, i.e., $\underline{u} \neq x$ and $\underline{u}Ex$. By transitivity of E, $\underline{u}Ez$. On the other hand, $\underline{u} \in (x) \subseteq (y)$, i.e. $\underline{u}Ey$. Therefore zOy. In conclusion, xMy, as required.

If $u \neq x$ we repeat the above argument with u instead of $\underline{u}$, reaching the same conclusion.

The above implication in the field of all preorders is not reversible. It can, however, be reversed for premereologies because for them it is equivalent to: $(x) \subseteq (y) \rightarrow xEy$ which, by (27), is reversible for any transitive frame.

Thus we obtain

(29) *Let E be premereological and x be complex. Then $xEy \leftrightarrow (x) \subseteq (y)$.*

Augmentation

6.16 The last equivalence fully deserves to be isolated as *the principle of augmentation*:

AP *If x is complex, then xEy iff $(x) \subseteq (y)$.*

Complexes grow with bodies!

6.17 It is noticeable that for asymmetric preorders the augmentation principle **AP** implies the following natural *principle of difference*:

DP *If x is complex, then $x \neq y$ iff $(x) \neq (y)$.*

Different complexes differ as to their bodies!

6.18 In the previous section we, in fact, proved:

(30) *For each premereology, the augmentation principle* **AP** *holds.*

Is it equivalent or weaker than premereology axiom, i.e., is **AP** equivalent to **ME**?

6.19 The answer both in general and in the case of preorders is: No, they are not equivalent.

As a matter of fact **AP** is weaker than **ME**. To see this consider two infinitely decreasing, reflexive and transitive chains:

$zE \dots x_2 E x_1 E x$, $zE \dots y_2 E y_1 E y$ with common point at infinity related to any point:

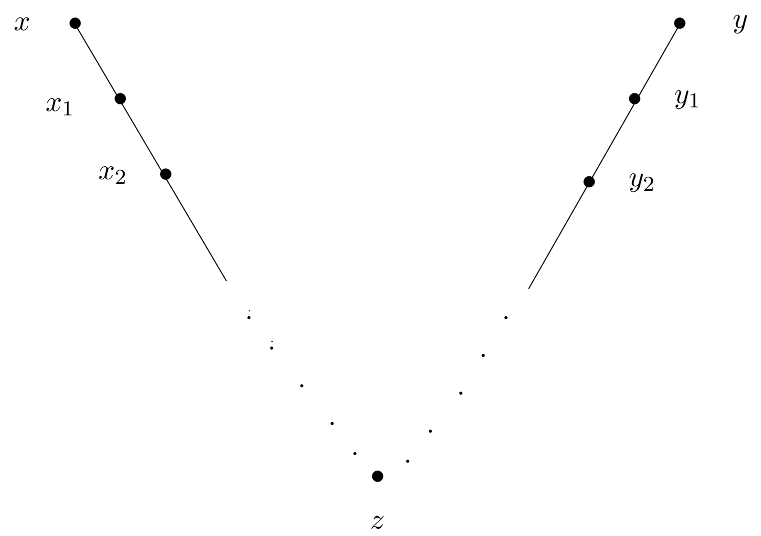


Fig. 2.

Clearly xMy, but $\neg(xEy)$. Hence **ME** doesn't hold. On the other hand, it is obvious that in our model **AP** is valid.

6.20 Note that each augmented structure must be a preorder when restricted to its complexes, but taken together with atoms (if any) it need be neither reflexive nor transitive. In general, the fields of augmented structures and preorders cross each other.

6.21 However, in a rather distinguished case augmentation is equivalent to the premereological characterization of "is". Namely, **AP** is equivalent to **ME** in inductive preorders.

A preorder is said to be *inductive* iff it fulfils *the Downard Chain Condition*:

DDC *Any chain of decreasing items is finite.*

In inductive preorders we can define by standard induction the *height* (or *rank*) for each object.

Observe that

(31) *Inductive preorders must contain atoms, and that any finite preorder is inductive.*

Now we are ready to establish the following theorem:

(32) *Let* $\langle U, E\rangle$ *be an inductive preorder. Then*
$\langle U, E\rangle$ *satisfies* **ME**, *i.e., it is a premereology iff*
$\langle U, E\rangle$ *satisfies* **AP**, *i.e., it is an augmented structure.*

Proof. The right hand implication follows by specification of (29).

For the reverse implication assume that $\langle U, E\rangle$ is an inductive preorder satisfying **AP**, but not **ME**.

Hence, for some x and y: xMy but $\neg(xEy)$. By (26) x is complex. Hence, by **AP**, $(x) \nsubseteq (y)$. Thus, for some z: $z \neq x$, zEx and $\neg(zEy)$.

We claim that zMy. To see this, let's take u such that uOz. By (24), uOx, for zEx. Hence uOy, for we supposed that xMy. Therefore, for any u, $uOz \rightarrow uOy$, i.e., zMy.

Thus we fond a such z that: $zPEx$, zMy and $\neg(zEy)$.

Clearly we can repeat the above procedure again and again, obtaining an infinite decreasing chain: $\ldots z_{i+1}Ez_i \ldots zEx$, which contradicts the inductiveness of $\langle U, E\rangle$.

6.22 Two corollaries follow:

(33) *For any finite preorder,* **ME** *is equivalent to* **AP**. *In other words, finite preorders are augmented iff they are premereological.*

This holds in virtue of (31) and (32).

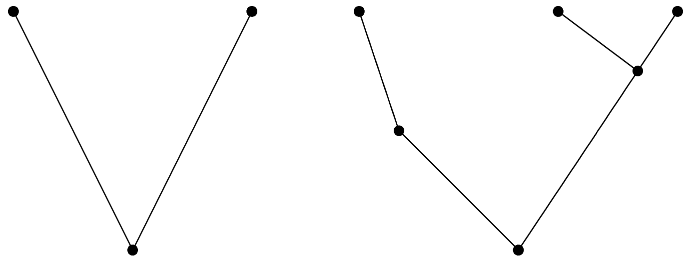
For the next corollary we need to recall: *A* (*reflexive*) *tree* is any preorder such that each of its elements has *at most one* immediate predecessor which is different from it.

Immediately from (32) we have

(34) *The converse of an inductive tree is premereological iff it is a linear preorder.*

The Scope of ME

6.23 Now, it is easy to see the power of the condensation axiom **ME**. Inter alia, it excludes all structures of the form



etc.

Fig. 3.

i.e. converses of all inductive trees different from chains.

For in premereology different complexes differ in their proper parts, which in proper mereology is strengthened to so-called supplementation principles, cf. 6.33.

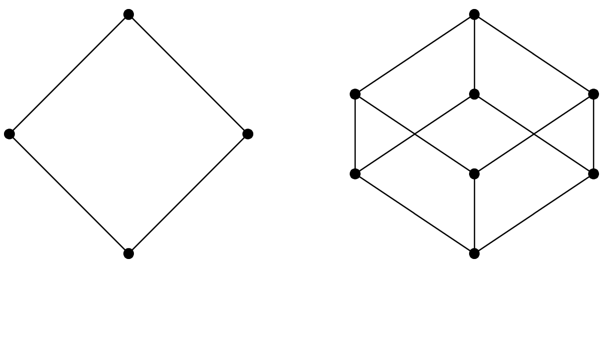
6.24 The axiom **ME** also excludes all atomic structures having the smallest element called *zero*, which are different from chains:

(35) *If $\langle U, E \rangle$ is an atomistic, inductive premereology with a zero element then it must be a chain.*

For in atomistic premereologies with zero all atoms, i.e. immediate successors of zero, collapse. Next, apply induction.

In the next section we will see that, in fact, zero trivializes premereologies.

Therefore the following structures are also excluded:



... Atomistic Boolean Algebras

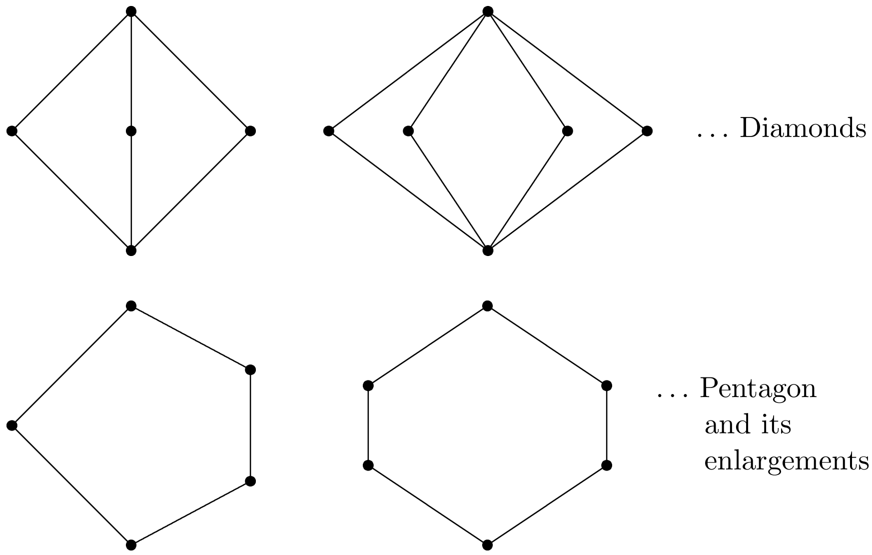


Fig. 4.

Notice that by cancelling zero in the above examples we obtain premereologies.

6.25 As a matter of fact, zero is forbidden in proper premereologies, for it trivializes them.

Indeed, if z is a zero, i.e., if $\forall x\ zEx$, then M is full: $\forall x \forall y\ xMy$. Hence, by **ME**, the starting relation E is also full.

If, in addition, we assume asymmetry condition **AS** then

(36) *Each asymmetric premereological structure $\langle U, E \rangle$ with zero is trivial, i.e., U is a singleton.*

6.26 To sum up:

1° Think that the ordered space $\langle U, E \rangle$ is *condensely* ordered[17] by E, if its points are connected by E only if their neighbours are also connected by E. Now, it is easy to see that that the condensation axiom **ME** realy does *condense* structures. In quite a lot of cases it condenses them to chains, or even to full relation structures. In noninductive cases it condenses them to fairly symmetric structures in which different objects can be differentiated from the bottom.

[17] Condense orderings discussed here should be distinguished from dense orders, i.e. relations enjoying the property that if xEy the for some z, $z \neq x$ and $z \neq y$ and $xEzEy$.

2° Preorders seem to catch quite well the formal properties of the relation "to be a part of". Premereologies, in addition, try to characterize wholes as objects having regular internal structures, which is expressed by the principle that complexes grow with their bodies.

This idea is clarified by means of the further mereological axioms.

Infinity

6.27 Let's return to the proof of the theorem (32).

This makes clear that for preorders the negation of the implication **AP**→**ME**, i.e. the conjunction **AP** ∧ ¬**ME**, expresses *infinity* in a way similar to the well-known formula of Schütte:

$$\forall x \exists y\ (xEy) \wedge \forall x \forall y \forall z\ (xEy \wedge yEz \rightarrow xEz) \rightarrow \exists x\ (xEx)$$

Schütte's formula is known to be false only in infinite domains, like the implication **AP**→**ME** we just have investigated.

To be more exact

(37) *Let* $\langle U, E\rangle$ *be a preorder falsifying* **AP**→**ME**, *i.e. verifying* **AP**∧¬**ME**. *Then* $\langle U, E\rangle$ *is infinite.*

6.28 The question "Does augmentation imply condensation?" is indeed very natural. However, to answer it in the negative we must refer to infinity.

Therefore, infinity seems to be deeply involved in the mereological approach to wholes.

Towards mereology

6.29 As it has been noticed previously, typical further mereological axioms clarify further the mereological structure of wholes.

In this sub-chapter I am going to overview the most standard axioms of this sort. I am, however, not going into details which are easy to find in Simons' and Libardi's books mentioned previously.

6.30 To guarantee ease of definitions people usually introduce the following *axiom-scheme of (mereological) comprehension :*

CA *For each formula* $A(x)$, *with a free variable* x:

$$\exists x\ A(x) \rightarrow \exists y \forall z\ (zOy \leftrightarrow \exists u\ (A(u) \wedge zOu)).$$

If A is satisfiable, then there exists y such that for any z, z overlaps with y iff it overlaps with some object satisfying A.

Clearly, assuming **AS**, y is the union (or fusion) of all objects satisfying A which overlap with z.

CA allows the introduction of all usual operations, including *intersection, union, complementation,* etc.

6.31 Observe that applying it to a tautologous formula we obtain

(38) $\exists y \forall z\ zOy$.

Hence, by **AS**, we deduce the existence of the universal object **1** such that $\forall z\ zO\mathbf{1}$. Therefore, by definition of M, $\forall z\ zM\mathbf{1}$. Finally, applying **ME**, we obtain

(39) $\forall x\ xE\mathbf{1}$.

We can also prove that

(40) *For any family of objects there exists its fusion.*

6.32 However, from the ontological point of view. both the existence of **1** as well as the last claim are very suspected.

Ontology deals with all possibilities. But quite a lot of collections seems to be mutually incompatible in such a way that they cannot coexist. Hence they have no fusion, which contradicts (40).

Therefore, I think, mereology with full comprehension deals with a rather special case in ontology. In its full power it, perhaps, is more useful in metaphysics (i.e. ontology of the world) or rather in the quasi-geometrical description of the world.

6.33 For the needs of ontology mereology has thereby to be weakened.

How far? Are premereologies better? Perhaps we should weaken them further?

6.34 Return to mereology. Another group of axioms is introduced to explain, step by step, the characterisation of difference provided by the Augmentation Principle. It includes, among other axioms, several supplementation principles:

The Weak Supplementation Principle:

WSP $\quad xPPy \to \exists z\ (zPPy \wedge zDSx)$

If x is a proper part of y then some proper part of y is disjoint from it.

The Strong Supplementation Principle:

SSP $$\neg(xPy) \rightarrow \exists z\ (zPx \wedge zDSy)$$

If x is not a part of y then some part of x is disjoint from y.

In the field of sets ordered by inclusion this supplementation is simply the set-theoretical difference $x - y$. This analogy is expressed in

The Remainder Principle:

RP $$\neg(xPy) \rightarrow \exists z \forall w\ (wPz \leftrightarrow wPx \wedge wDSy)$$

If an object is not part of a second object, then there exists a unique remainder (difference) of the first minus the second[18].

6.35 All these axioms are introduced in order to increase the similarity of mereologies to Boolean structures.

Fine. But should our ald acquaintance with non-boolean situations be forgotten in metaphysics?

Summary

6.36 In the present overview of mereology two new classes of structures has been introduced and investigated: premereologies and augmented structures. They both extend mereology, in two slightly different ways, however.

6.37 The relations between types of structures investigated in the present chapter are as follows:

Each mereology is a premereology, which, in turn, is at once a preorder and an augmented structure. Classes of preorders and augmented structures cross each other.

Using self-explaining symbols: **MS** — for mereologies, **PS** — for premereologies, **POS** — for preorders and **AS** - for augmented structures, these connections can be drawn as follows:

[18] Cf. Simons [25]

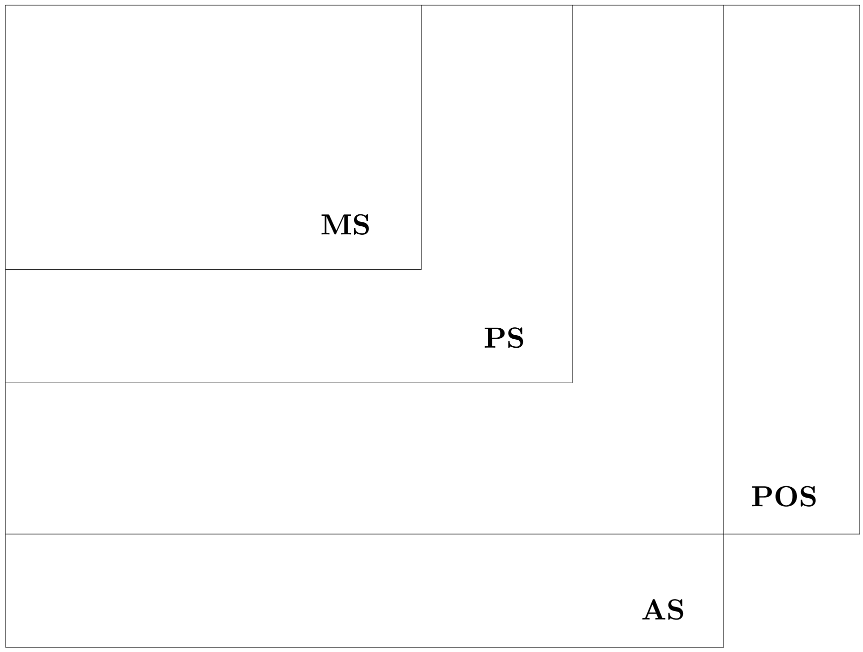


Fig. 5.

7. LEŚNIEWSKI'S ONTOLOGY

7.1 No survey of available theories of ontologically relevant orders can omit Leśniewski's Ontology[19]. It should interest us particularly because Leśniewski constructed it with a strong ontological idea in mind — to grasp the most general and primitive, the deep sense of the verb "is".

Whether, and to what extent, he was successful is a problem for discussion. In particular, it is not clear in which way the formal machinery of Leśniewski's Ontology expresses and is connected with his never published nominalistic philosophy which is known from tradition.

These questions call for attention. They need subtle and extended discussion. Because of shortage of space, here I can only announce it. An interested reader is referred to an accompanying paper "The Nominalism of Leśniewski's Ontology" [18], cf. also Smith [26].

[19] Cf. Leśniewski [8] and [9]. Cf. also Srzednicki i Rickey [27].

In this chapter, I am only discussing Leśniewski's axiom in relation to the systems introduced above.

7.2 Leśniewski's axiom is as follows:

LON $xEy \leftrightarrow (\exists z\ zEx) \wedge \forall z \forall u\ (uEx \wedge zEx \rightarrow uEz) \wedge \forall z\ (zEx \rightarrow zEy)$

This is, to be sure, a rather long and somehow obscure formula. Usually, the axiom is treated as an implicit definition of singular inclusion. The three conditions on its right-hand side correspond, in a sense, to Russell's condition for definite descriptions: there is at least one x, there is at most one x, and wherever is x is y[20].

This rather obvious association can, however, be misleading. Russell's conditions are expressed for a fixed predicate (or formula) which occurs in each of the three conditions involved. This, however, does not hold in our case. On the other hand, there is no straight path from Leśniewski's basic expression "xEy" to the atomic formula of classical logic "$y(x)$".

Leśniewski's axiom should thereby be understood not through its supposed genesis, but through logical analysis.

7.3 First of all, the form of the axiom is similar to the one pointed out in the case of preorders and premereologies. Their axioms, we remember, are equations indicating suitable equivalents to the primitive relation E, respectively $E = P$ and $E = M$.

Which equation is appropriate for the case of **LON**?

Observe that all three formulas on the right-side of **LON** are old friends. The third one is our definition of the part-relation. Therefore, Leśniewski's axiom is of the form $E = P \cap (S \times U)$, where S is a *singularity condition*:

$S(x)$	$:= N(x) \wedge Sol(x)$, where	
$N(x)$	$:= \exists z\ zEx$	*Nonemptiness Condition*, and
$Sol(x)$	$:= \forall u \forall z\ (uEx \wedge zEx \rightarrow uEz)$	*Uniqueness Condition*[21]

7.4 Observe that in Leśniewski's ontologies

(41) $xEy \rightarrow xPy$

Hence, by (4), E is transitive. In general, however, it is not reflexive. Hence Leśniewskian ontologies *en gros* are not preorders. As a matter of

[20] Cf. Simons [25].

[21] In the theory of relations this is known as the *anti-Euclidean condition.*

fact, the realm of preorders and the realm of Leśniewskian ontologies cross each other. They cross also with the realm of augmented structures.

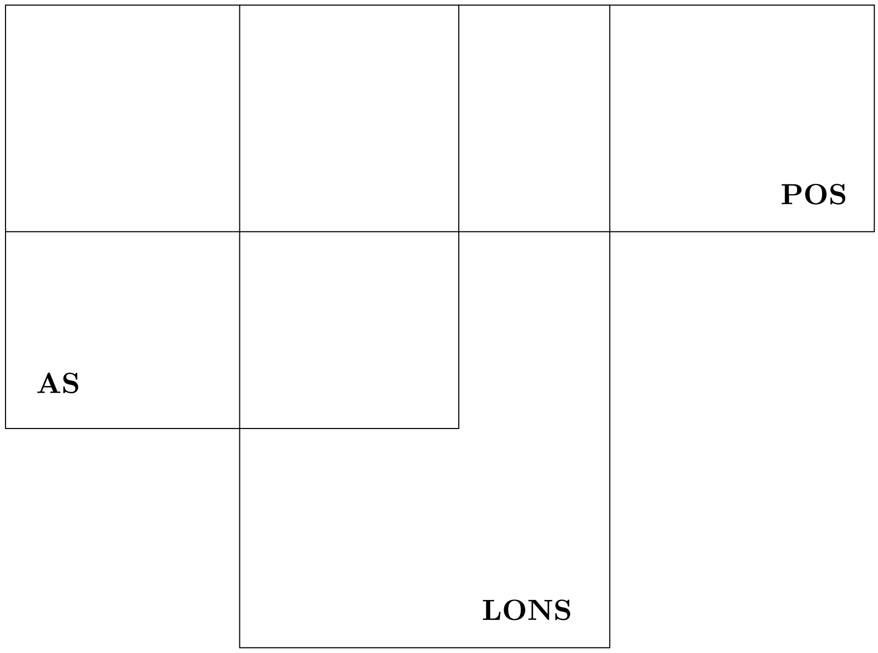


Fig. 6.

7.5 What condition, then, together with transitivity, characterizes Leśniewski's Ontology?

We can find several conditions, including

(42) **LON** *is equivalent to:* $\mathbf{T} \wedge (S(x) \leftrightarrow \mathrm{Id}(x)) \wedge (xEy \rightarrow \mathrm{Id}(x))$, where $\mathrm{Id}(x) := xEx$

For the proof, explication and further discussion of Leśniewski's Ontology the reader is referred to [18].