

# IMPLICATION ZROUPOIDS: AN ABSTRACTION FROM DE MORGAN ALGEBRAS

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## ABSTRACT

In 1934, Bernstein gave a system of axioms for Boolean algebras in terms of implication only (see [1]); however, his original axioms were not equational. A quick look at his axioms would reveal that, with an additional constant, they could easily be translated into equational ones. In 2012, this (modified) Bernstein's theorem was extended to De Morgan algebras in [9]. Indeed, it is proved there that the varieties of De Morgan algebras, Kleene algebras, and Boolean algebras are term-equivalent to varieties whose defining axioms use only the implication  $\rightarrow$  and the constant 0.

These results motivated me to introduce, in [9], a new equational class of algebras called "Implication zroupoids" in [9].

An algebra  $\mathbf{A} = \langle A, \rightarrow, 0 \rangle$ , where  $\rightarrow$  is binary and 0 is a constant, and  $x' := x \rightarrow 0$ , is called an *implication zroupoid* if  $\mathbf{A}$  satisfies:

$$\begin{aligned} \text{(I)} \quad & (x \rightarrow y) \rightarrow z \approx [(z' \rightarrow x) \rightarrow (y \rightarrow z)]', \\ \text{(I}_0\text{)} \quad & 0'' \approx 0. \end{aligned}$$

During the last two years, Juan Cornejo and I have continued these investigations into the structure of the lattice of subvarieties of the variety of implication zroupoids in [3], [4], [5], [6] and [7].

In this talk I would like to survey some of our results on implication zroupoids.

## REFERENCES

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